CONVECTIVE WALL PLUME

V. D. Zimin and Yu. N. Lyakhov

We study laminar convective flow along a vertical flat plate from a horizontal line heat source located on the plate surface. The problem is solved numerically in the boundary-layer approximation for an incompressible fluid and without account for heat transfer at the fluid-plate boundary. The results of the numerical solution are compared with experiment. The study of the temperature field from the heated horizontal wire located on the surface of a vertical plate made from foam plastic was conducted using the IAB-451 shadow instrument by the diffraction interferometer method.

NOTATIONS

x and y = vertical and horizontal coordinates measured from the line heat source; x° = vertical coordinate measured from the center of the heated horizontal wire; u and v = vertical and horizontal velocities; T = temperature reckoned from the undisturbed fluid temperature, taken as zero; T_w = wall temperature; B = complex defined by the physical properties of the fluid; ν = kinematic viscosity; χ = thermal diffusivity; β = volumetric expansion coefficient; c_p = specific heat at constant pressure; ρ = density; g = gravitational acceleration; q = heat flux density through horizontal section of plume (in cal/cm · sec); q_l = specific thermal power released by the heated wire (in cal/cm · sec); t = dimensionless temperature profile function; ξ = dimensionless variable.

1. Let us examine in the boundary-layer approximation the problem of free convection from a horizontal line heat source located on a vertical plate with zero thermal conductivity. The equations of the convective boundary layer

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta T$$
$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\chi\frac{\partial^2 T}{\partial y^2},\quad \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1.1}$$

with the boundary and integral conditions

$$u=v=\frac{\partial T}{\partial y}=0\quad(y=0)$$
$$u=T=0\quad(y=\infty)$$
$$\rho c_p\int_0^\infty uT\,dy=\mathrm{const}\,(x)\equiv q \tag{1.2}$$

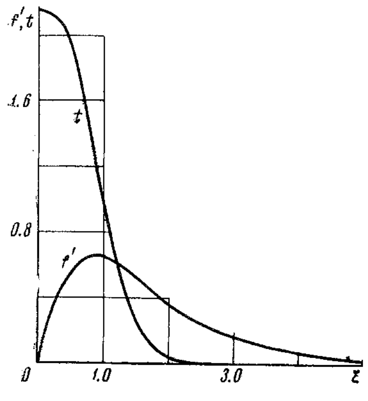

Fig. 1

admit, as in the case of a source in an infinite medium [1], the similarity transformations

$$u=\left[\frac{qg\beta}{\rho c_p\sqrt{\nu}}\right]^{2/5}x^{1/5}f'(\xi),\qquad v=\frac{1}{5}\left[\frac{qg\beta\nu^2}{\rho c_p}\right]^{1/5}x^{-2/5}\{2\xi f'(\xi)-3f(\xi)\}$$
$$T=\left[\frac{q^4}{\rho^4 c_p^4 g\beta\nu^2}\right]^{1/5}x^{-3/5}t(\xi),\qquad \xi=\left[\frac{qg\beta}{\rho c_p\nu^3}\right]^{1/5}yx^{-2/5} \tag{1.3}$$

The transformations (1.3) reduce (1.1) to a system of ordinary differential equations

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$$f''' + {}^3/_5 f''f - {}^1/_5 (f')^2 + t = 0, \quad t' + {}^3/_5 Ptf = 0 \tag{1.4}$$

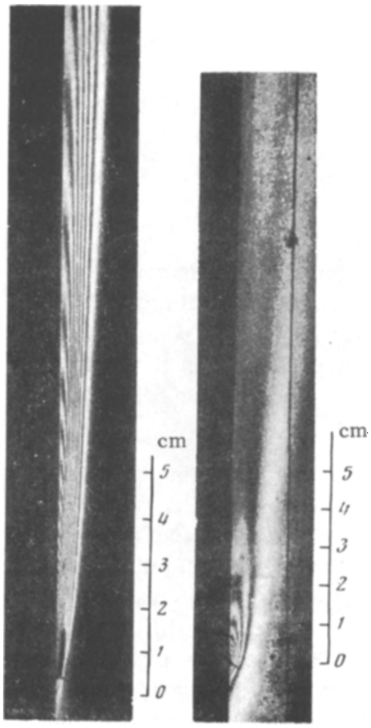

Fig. 2

Equations (1.4) were solved numerically for P = 7 with the boundary conditions

$$f = f' = 0 \quad (\xi = 0), \qquad f' = t = 0 \quad (\xi = \infty) \tag{1.5}$$

The solution was carried out by the method of finite difference with the integral-norming condition

$$\int_0^\infty f'td\xi = 1 \tag{1.6}$$

The temperature and vertical velocity distributions obtained as a result of the numerical solution are shown in Fig. 1.

2. The experimental study of the temperature field in the wall convective plume was carried out using a diffraction interferometer on the setup and using the technique described in [2]. The heat source was a dc-heated 0.195-mm-diam platinum wire 20.0 cm long. The wire was positioned horizontally along the optical axis of the setup on the surface of a vertical plate with dimensions 20 × 30 × 5 cm, fabricated from fine-pore foam plastic with highly finished surface. The experiments were conducted with distilled water at room temperature (Prandtl number P = 7). Since the thermal conductivity of the porous plastic is 0.7-1 cal/cm · sec · °C [3], the relative thermal conductivity $\varkappa_0$ of the fluid relative to the wall was about 20, which corresponded sufficiently well to the condition adopted above on the absence of heat transfer at the boundary.

An interferogram of the observed temperature field of the wall convective plume is shown in Fig. 2a. For comparison Fig. 2b shows the interferogram for the case $\varkappa_0 = 0.6$ (air-foam plastic). In the latter case there is clearly marked downward curvature of the isotherms near the wall, which indicates the presence of considerable heat flux from the fluid to the wall.

The interferograms of Fig. 2a were analyzed using the technique adopted in [2]. The experimental results were used to plot the dependence of the quantity

$$\eta \equiv \left[\frac{q_l^{4/5}}{BT_w}\right]^{5/3} = [t(0)]^{-5/3} x, \qquad B = (g\beta\nu^2\rho^4 c_p^{\,4})^{1/5} \tag{2.1}$$

on the vertical coordinate x°, measured from the center of the heated wire. Here, in place of the quantity q used in (1.2) and (1.3) we used the thermal power q_l released per unit wire length.

The results for several thermal regimes are shown in Fig. 3a, where the points 1, 2, 3, 4 correspond to the values $q_l = 14.4 \cdot 10^{-3}$, $20.0 \cdot 10^{-3}$, $28.5 \cdot 10^{-3}$, $37.8 \cdot 10^{-3}$ cal/cm · sec. The straight line drawn through the experimental points by the method of least squares is described by the equation $\eta = -0.112 + 0.422\ x°$. The mean-square error in the free term amounts to 15%, and that in the slope is 0.2%. The slope of the straight line was used to calculate the maximal value of the dimensionless temperature profile function t(0) = 1.67. Comparison of the result with the theoretical value t(0) = 2.16 yields the relation $q = 0.73\, q_l$.

In view of the fact that q also appears in the expression for the dimensionless variable ξ, we made a comparison of the theoretical and experimental results on the basis of the thickness of the thermal boundary layer. It follows from (1.3) that the quantity

$$\zeta \equiv (ky)^{5/2} = (\xi)^{5/2} x, \qquad k = \left[\frac{qg\beta}{\rho c_p \nu^3}\right]^{1/5} \tag{2.2}$$

is proportional to the vertical x. For a given value of the vertical coordinate x° we found that distance $y_{1/2}$ from the plate at which the temperature was half that at the wall. The dependence of $\xi_{1/2}$ on x° is shown in

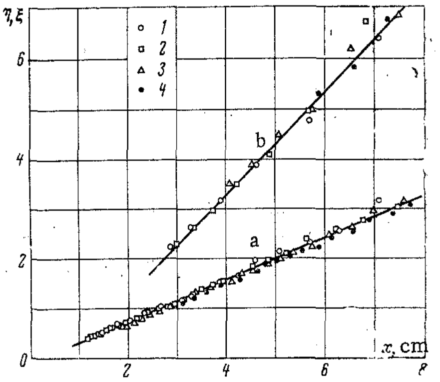

Fig. 3

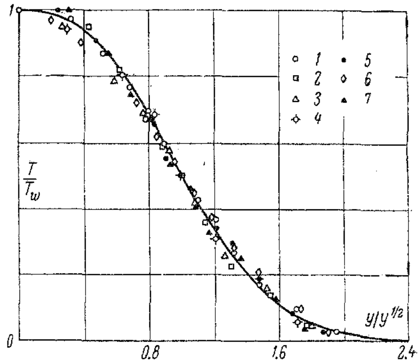

Fig. 4

Fig. 3b. Here again, in place of q in (2.2) we used the value of q_l. Using the method of least squares, this dependence can be expressed by the equation

$$\zeta_{1/2} = -0.83 + 1.04x^{\circ}$$

with a mean-square error in the free term of 15% and 2% in the slope. The slope of the straight line was used to calculate the value of $\xi_{1/2}$. Comparison of this result with the value $\xi_{1/2} = 0.95$, obtained on the basis of numerical calculation (Fig. 1), again leads to the relation $q = 0.73\ q_l$.

Thus the experimental and numerical solution results coincide if we consider that only part of the power released by the heated wire goes into formation of the wall plume. The thermal power losses can be explained in part by heat transfer into the depth of the plate in the zone immediately adjacent to the source. In this zone, in spite of the relatively small thermal conductivity of the plate material, the heat losses may be significant because of the large temperature gradient. A second reason for the lack of complete agreement between the theoretical and experimental constants in the similarity transformations may be inflow of cold fluid to the source from regions located below its level, which is not taken into account in the theoretical examination of the problem in the boundary-layer approximation.

Figure 4 shows the dimensionless temperature profiles in the wall plume for two thermal regimes. The experimental points 1, 2, 3, 4 correspond to sections at distances 4, 6, 8, 10 cm from the center of the heated wire for $q_l = 20.0 \cdot 10^{-3}$ cal/cm · sec; points 5, 6, 7 correspond to sections 8, 10, 12 cm and $q_l = 37.8 \cdot 10^{-3}$ cal/cm · sec. The solid curve represents the temperature profile obtained on the basis of the numerical solution of the problem. We see from the figure that the theoretical and experimental temperature profiles coincide to within experimental error.

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LITERATURE CITED

1. Ya. B. Zel'dovich, "Limiting laws of freely rising convective flows," Zh. Éksperim. Tech. Phys., 7, No. 12 (1937).
2. Yu. N. Lyakhov, "Experimental study of free convection above heated horizontal wire," PMTF (J. Appl. Mech. and Tech. Phys.), 11, No. 2 (1970).
3. G. W. C. Kaye and T. H. Laby, Tables of Physical and Chemical Constants, Wiley (1959).